Weakly coupled Pfaffian state as a type 1 quantum fluid

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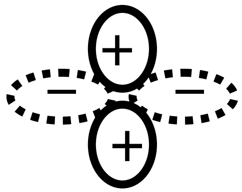
Superconducting order parameter

$$\Delta(\mathbf{r},\mathbf{r}') = g(\mathbf{r}-\mathbf{r}')\langle\psi(\mathbf{r})\psi(\mathbf{r}')\rangle, \quad \Delta(\mathbf{r}-\mathbf{r}') = \int d\mathbf{k}\,\Delta(\mathbf{k})\,e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}$$

in S-wave superconductors $\Delta(\mathbf{r}=\mathbf{r}') = \Delta(\mathbf{r})$

in D- and P-wave superconductors $\Delta(\mathbf{r}=\mathbf{r}') = 0$

D-wave (singlet) order parameter
can be chosen as real



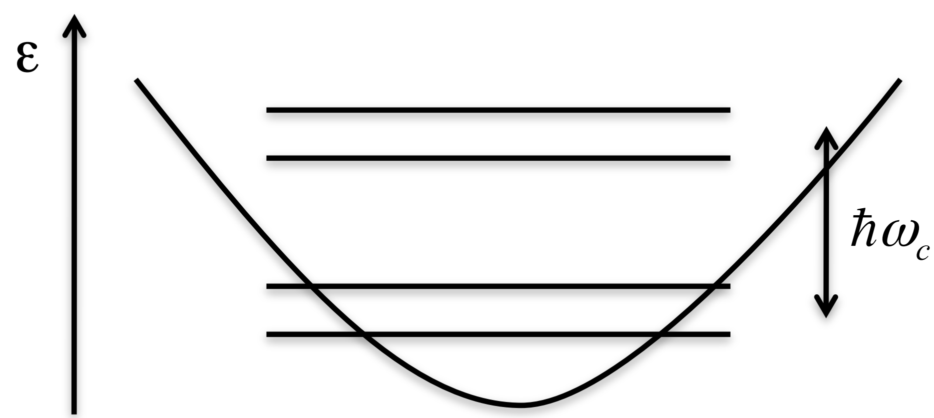
P-wave (p+ip)
order parameter

$$\Delta(p) \propto p_x + ip_y$$

$$p_x^2 + p_y^2 = p_F^2$$

the system has an isotropic gap

Spectrum of electrons in magnetic field



J. Jain, R. Laughlin, S. Girvin, A. McDonnald,
S. Kivelson, S.C. Zheng, E. Fradkin, F. Wilczek,
P. Lee, N. Read, G. Moore, B. Halperin, D. Haldane

Chern-Simons theory of quantum Hall effect (Fermion version k=2)

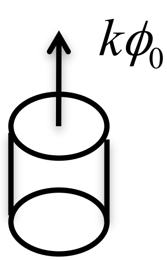
$$\nabla_j \to \nabla_j - i\frac{e}{\hbar c}\mathbf{a}(\mathbf{r}_j), \quad \mathbf{a} = k\frac{\hbar c}{e}\nabla_j \sum_{j\neq k}\alpha_{jk}, \qquad \alpha_{jk} = \operatorname{Im}\ln(z_j - z_k)$$

$$\mathbf{b}_j = \nabla_j \times \mathbf{a}_j = k\phi_0 \sum_{i\neq k}\delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$\phi_0 = \frac{hc}{e}$$

b and **a** are statistical magnetic field and vector potential.

The statistical phase can be interpreted as an Aharonov-Bohm effect: when charge is moving around the flux ($\kappa\phi_0$) it acquires a phase $\kappa\pi$



Halperin- Lee -Read (HLR) state:
“Fermi liquid” of composite Fermions, k=2

At the filling factor ν=½ the statistical and the external magnetic fields cancel each other **B+b =0**, and on the mean field level the system is in a Fermi liquid state without magnetic field.

$$\frac{1}{m} \approx \frac{e^2 L_H}{\hbar^2}; \qquad E_F \approx \frac{e^2}{L_H} \approx \frac{\hbar^2}{m L_H^2}$$

Mean field electrodynamics of HLR state

$$\mathbf{b} = k\phi_0\left(\rho(\mathbf{r}) - \rho_{1/2}\right), \qquad -(k\phi_0 / c)\, j_i = \varepsilon_{ij} e_j$$

Ohm’s law for composite Fermions

$$\mathbf{j} = \sigma\left(\mathbf{E} + \mathbf{e}\right)$$

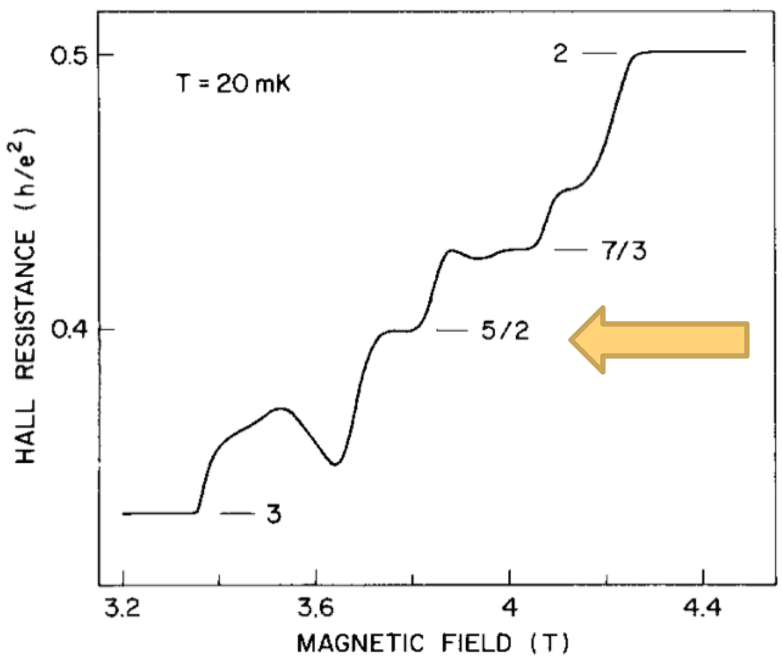


Fig. 2. Hall resistance ρ_{xy} at 20 mK in the region around $\nu = \frac{5}{2}$.

J.P. Eisenstein, R.L. Willet, H.L. Stormer, L.N. Pffiffer, K.W. West

More-Read Pfaffian 5/2 QH state, weakly coupled (BCS) P-wave superconductivity of composite fermions

$$\Delta(\mathbf{k}) \propto (p_x \pm i p_y), \qquad p_x^2 + p_y^2 = p_F^2$$

Mean field Gorkov equations

$$\left(i\hbar\partial_0 - e\left(\mathrm{a}_0 + \mathrm{A}_0\right)\right)\Psi + \frac{\hbar^2}{2m}\left\{-i\nabla - \left(\mathbf{a} + \mathbf{A}\right)\right\}^2\Psi + \Delta(\mathbf{r},\mathbf{r}')\Psi^*(\mathbf{r}') = 0$$

$$\Delta(\mathbf{r},\mathbf{r}') = g(\mathbf{r}-\mathbf{r}')\langle\Psi(\mathbf{r}')\Psi(\mathbf{r})\rangle$$

$$\mathbf{b} = \nabla\times\mathbf{a} = -k\phi_0\langle\Psi(\mathbf{r})\Psi(\mathbf{r})\rangle$$

$$\mathbf{z}\times\mathbf{e} = z\left(\partial_0\mathbf{a} - \nabla a_0\right) = -k\phi_0\mathbf{j}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{2mi}\left[\Psi(\mathbf{r})\left(\nabla\Psi^*(\mathbf{r})\right) - \Psi^*(\mathbf{r})\left(\nabla\Psi(\mathbf{r})\right)\right] - \frac{e}{mc}\left(\mathbf{a} + \mathbf{A}\right)\Psi\Psi^*$$

z is a unit vector perpendicular to the plane

Correspondence between the perfect conductivity of the superconductors and the quantization of the Hall conductance:

Meissner effect—--incompressibility

Quantized vortices------fractionally charged quasiparticles

$$\Delta = |\Delta| e^{i\chi}$$

$$2\phi_0 j_i = \varepsilon_{ij} e_j$$

$$\mathbf{j} = N_s \mathbf{v}_s; \quad \mathbf{v}_s = \frac{\hbar}{2m}\left(\nabla\chi - \frac{2e}{\hbar c}\left(\mathbf{A}+\mathbf{a}\right)\right)$$

$$0 = m\frac{\partial \mathbf{v}_s}{\partial t} = e(\mathbf{E}+\mathbf{e}) = 0$$

$$\sigma_{xy} = \frac{1}{2}\frac{e^2}{\hbar}, \quad \frac{5}{2}\frac{e^2}{\hbar}, \quad \ldots\ldots.$$

$$e^* = e\int d\mathbf{r}\,|\Psi|^2 = \frac{1}{k\phi_0}\oint \mathbf{a}\,d\mathbf{r} = \frac{e}{k\phi_0}\frac{\phi_0}{2} = \frac{e}{4}$$

Two types of conventional superconductors

$$\lambda_L = \left(\frac{mc^2}{n_e e^2}\right)^{1/2} = \frac{1}{(n_e a_B)^{1/2}\alpha}$$

$$if\ \ r_s \approx 1, \quad \lambda_L \approx \frac{1}{n_e^{1/3}\alpha}$$

$$\xi = \frac{v_F}{|\Delta|}$$

a_B is the Bohr radius

n_e is the electron concentration

α is the fine structure constant

$\Delta << \mathrm{E_F}$ is the gap

Type 1: $\lambda < \xi$ surface energy is positive

Type 2: $\lambda > \xi$ surface energy is negative

Two characteristic lengths in the pfaffian state

1. Coherence length $\xi = \dfrac{v_F}{|\Delta|}$

2. Penetration length of "the statistical magnetic field" $\lambda \approx L_H$

L_H is the magnetic length

Two characteristic energy scales

The gap $|\Delta|$

The Fermi energy $E_F \approx \dfrac{\hbar^2}{m L_H^2}$

Two possible types of quantum Hall fluids

a) Type 2 QH fluids where roughly $\xi \sim \lambda$.
In this case the surface energy between HLR and Pfaffian states is negative. Consequently density deviations are accommodated by the introduction of single quasiparticles/votices

b) Type I QH state: $\xi >> \lambda$, (or $E_F >> \Delta$)
In this case the surface energy between is positive. Quasiparticles (vortices) agglomerate and form multi-particle bound states

electronic microemulsions

How do we know that in the Pfaffian state $\xi \gg \lambda$?

1.Numerical simulations:

H. Lu, S. das Sarma, K. Park, cond-mat. 1008.1587;

P. Rondson, A. E. Feiguin, C. Nayak, cond. mat. 1008.4173;

G. Moller, A. Woijs, N. Cooper, cond-mat. 1009.4956

$$\xi/\lambda \sim 10-30$$

2.a) Activation energy in transport experiments approximately two orders of magnitude smaller than E_F . Sometimes decreases further as a function of gate voltage and parallel magnetic field.

b) the characteristic temperature where 5/2 plateau of QHE disappears is much smaller then E_F

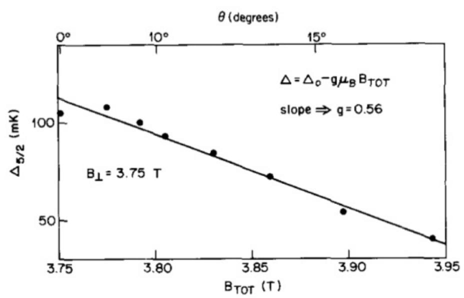


Fig. 4. Energy gap Δ versus total magnetic field B_{tot}. Straight line gives g-factor $g=0.56$.

J.P. Eisenstein, R.L. Willet, H.L. Stormer, L.N. Pffiffer, K.W. West 1990

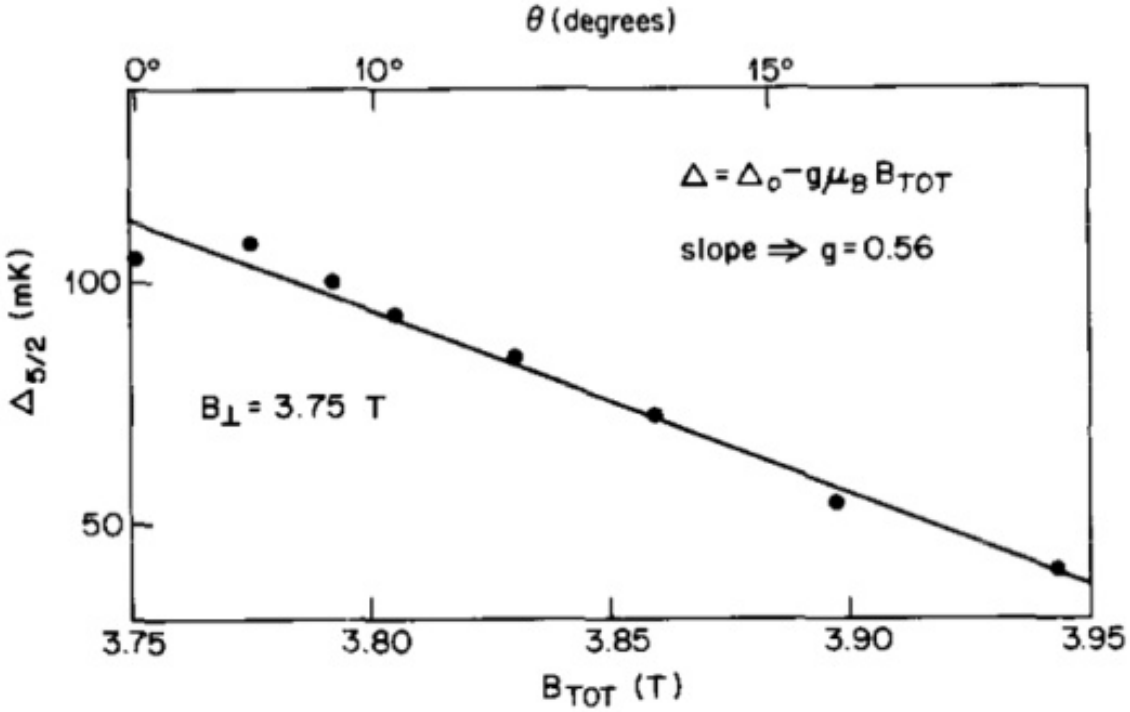


Fig. 4. Energy gap Δ versus total magnetic field B_{tot}. Straight line gives g-factor $g=0.56$.

$$E_F \approx 100K$$

If ξ>>λ vortices agglomerate into big bubbles

$$\text{Condensation energy of a votex} \approx \mu\Delta^2 \times \xi^2 \approx E_F \qquad (\xi \propto \frac{1}{\Delta})$$

$$\text{Coulomb energy associated with merging of two votices} \approx \frac{e^2}{\xi} << E_F$$

$$\Sigma \approx \frac{|\rho_e - \rho_{5/2}|}{N_c}\left(E_F + \frac{e^2 N_c^2}{\xi}\right)$$

$$N_c \approx \left(\frac{E_F \xi}{e^2}\right)^{1/2} \approx \left(\frac{\xi}{\lambda}\right)^{1/2} \propto \frac{1}{|\Delta|^{1/2}} >> 1$$

N_c is the number of electrons in the bubble

If $N\xi^2$ ~1 the system is in "electronic microemulsion phase" which can be visualized as a mixture of HLR and Pfaffian on mesoscopic scale.

Schematic phase diagram

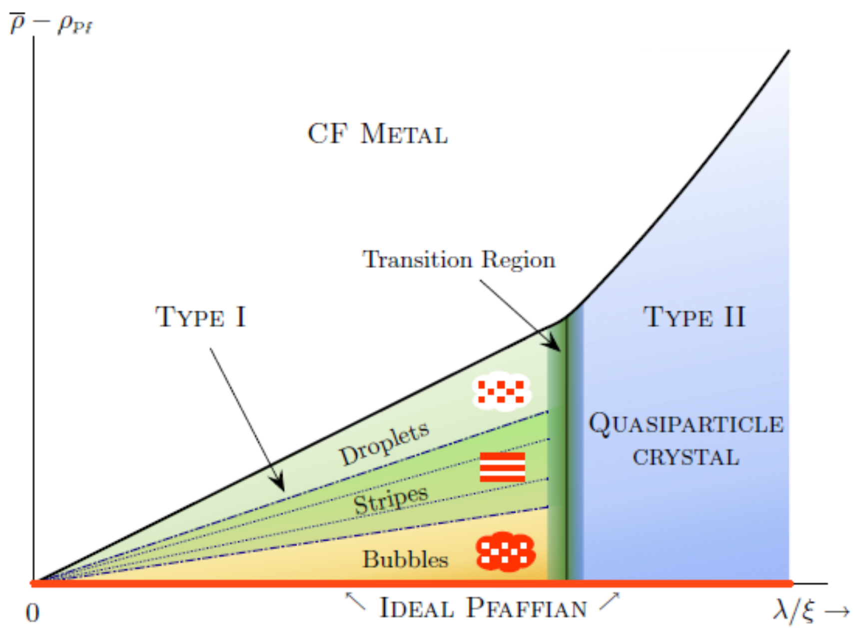


FIG. 1: Schematic phase diagram of the Pfaffian phases with fixed Coulomb interactions as a function of density deviation from $\nu = 5/2$ (pseudomagnetic field) and the Landau-Ginzburg parameter, λ/ξ. Typical configurations of the different inhomogeneous phases are shown, with red representing the Pfaffian and white the metallic phase. For short-ranged interactions, the microemulsion phases in the Type I region are replaced by a phase-separated intermediate state, with the Pfaffian fraction decreasing continuously to zero as the boundary to the composite fermion metal is approached.

D. Scalapino, M. Tinkham, M. Schmidt, G. Schoen, V. Volkon, A. Artemenko, A. Aronov, V. Gurevich

hydrodynamics of the Pfaffian state. Penetration of electric field into the system of superconducting composite Fermions

$$\Delta = |\Delta| e^{i\chi}$$

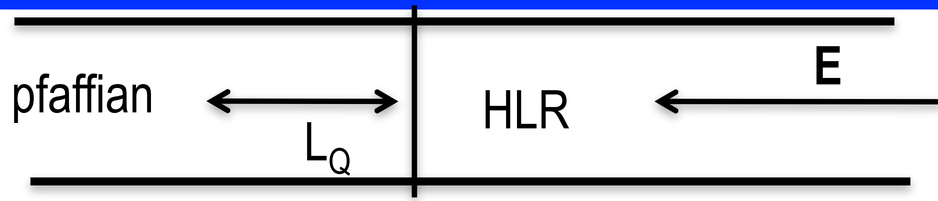
$$2\phi_0 j_i = \varepsilon_{ij} e_j$$

$$\mathbf{j} = N_s \mathbf{v}_s + \boxed{\sigma \nabla\Phi}; \quad \mathbf{v}_s = \frac{\hbar}{2m}\left(\nabla\chi - \frac{2e}{\hbar c}(\mathbf{A} + \mathbf{a})\right)$$

$$0 = m\frac{\partial \mathbf{v}_s}{\partial t} = e(\mathbf{E} + \mathbf{e}) + \boxed{\nabla\Phi}; \quad \boxed{\Phi = \frac{\hbar}{2}\frac{\partial \chi}{\partial t} + \left(A_0 + a_0\right)}$$

$$\boxed{D\Delta\Phi = -\frac{\Phi}{\tau_Q};} \quad \tau_Q \to \infty \text{ at the critical point}$$

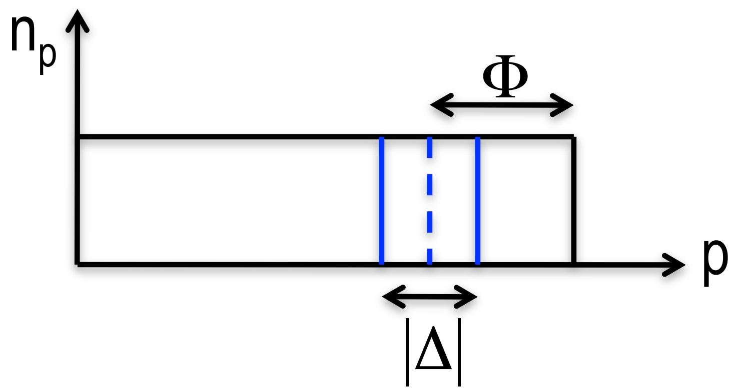
Proximity effect at the HLR-pfaffian states boundary



$$k\phi_0 j_i = \varepsilon_{ij} e_j; \qquad \mathbf{j} = N_s \mathbf{v}_s + \sigma \nabla \Phi;$$

$$e(\mathbf{E} + \mathbf{e}) + \nabla \Phi = 0; \qquad D\Delta\Phi = -\frac{\Phi}{\tau_Q}; \qquad L_Q = \left(D\tau_Q\right)^{1/2}$$

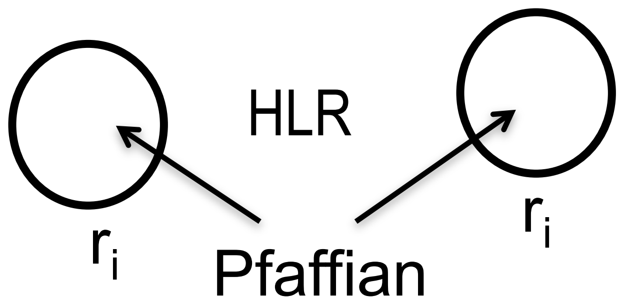
Transition between the HLR and Pfaffian state Ohm's law takes place over the distance of order L_Q , which diverges at the critical point



Φ is the imbalance of populations of electron and hole branches of spectrum

$$\tau_Q \approx \tau_\varepsilon \frac{T}{|\Delta|} >> \tau_\varepsilon \qquad if \qquad |\Delta| << T$$

Elastic electron scattering destroys p-wave superconductivity when the electron mean free path $l \sim \xi$ becomes of order of the superconducting coherence length. A generic feature of the disordered quantum transitions is that the characteristic inter-puddle distance is much bigger than their characteristic size.



a criterion of a quantum (T=0) phase transition:

$$X_{ij} = \chi_i \chi_j J_{ij} J_{ji} \approx 1$$

χ_i is the susceptibility of a puddle
J_{ij} is the Joshepson coupling between puddles
χ_ι and J_{ij} are random quantities

Susceptibility of a puddle

R~Rc

$$\chi = e^{G_{fff}} \qquad 3D \qquad \text{Kosterlitz}$$

$$\chi = e^{\sqrt{G_{2D}}} \qquad 2D, \qquad \text{Feigelman, Larkin, Skvortsov}$$

G_{eff} and G_{2D} are conductances of a cube of HLR metal of size R, and 2D film respectively

susceptibility is an exponential function of G >>1

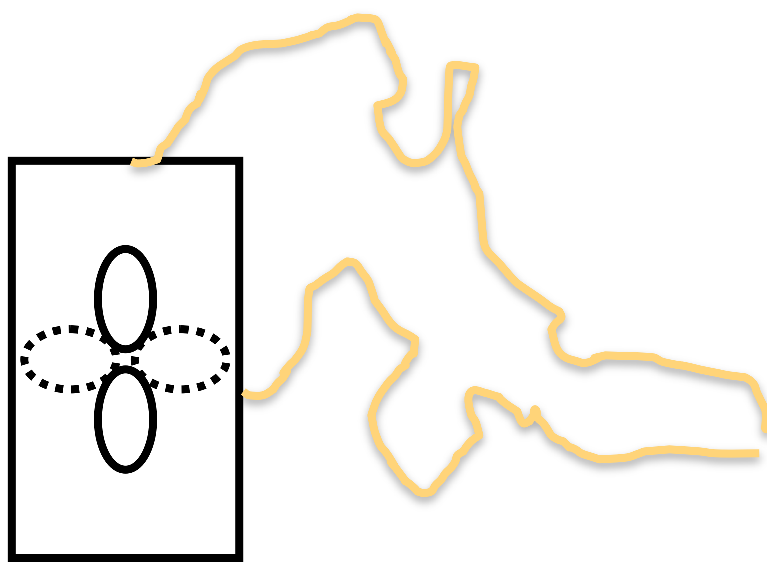
Inter-puddle joshepson coupling

At T=0 Josephson coupling between the puddles decay with the inter-puddle distance slowly.

$$J_{ij} \propto \frac{1}{r_{ij}^{x}}$$

consequently, at the point of superconductor-metal transition the distance between the puddles is parametrically bigger than their size.

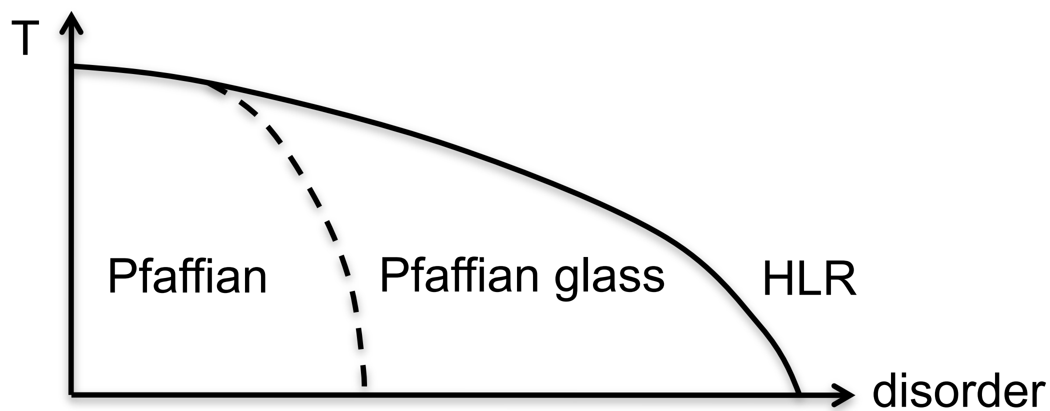
In disordered systems at large distances the sign of the Joshepson coupling is random.



An effective model of Joshepson junctions

$$\Sigma = \sum_{i \neq j} J_{ij} \cos\left(\chi_i - \chi_j\right) + \text{quantum kinetic energy}$$

Since J_{ij} have random sign, near the critical point the system is Pfaffian (p-wave superconducting) glass



Conclusion:

Weakly coupled Pfaffian state is equivalent to Type 1 p+ip superconducting state.
In this state vortices attract each other and agglomerate into big bubbles. There is a quantum phase transition between HLR and Pfaffian states as a function of disorder